Mark Scheme (Results)

June 2011

AEA Mathematics (9801)

June 2011
Publications Code UA028410

June 2011
9801 Advanced Extension Award Mathematics
Mark Scheme

Question	Scheme	Marks	Notes
1.	$\frac{\sin(\theta+35)}{\cos(\theta+35)}=\frac{\cos(\theta-53)}{\sin(\theta-53)}$	M1	Use of correct defns for tan and cot
	$0 = \cos(\theta-53)\cos(\theta+35)-\sin(\theta+35)\sin(\theta-53)$ $0 = \cos(2\theta - 53 + 35)$	M1	Use of cos(A+B) rule to reach single trig function
	$2\theta - 18 = 90, 270$ so **x = 54, 144**	A1A1 **(4)**	A1 for 54 and A1 for 144
	Use of tan ($A \pm B$) doesn't score until tan2θ= tan(90 - -18)		
ALT	$\tan(\theta+35)=\tan[90-(\theta-53)]$ $\theta+35=90-(\theta-53)$ or $\theta+35=90-(\theta-53)+180$	M1 M1	Use of cot$x$ = ± tan(90±x) either
2.	$\left(1+\tan\frac{1}{2}x\right)^2=1+2\tan\left(\frac{1}{2}x\right)+\tan^2\left(\frac{1}{2}x\right)$	M1	Attempt to multiply 3 terms at least 2 correct
	$=\sec^2\left(\frac{1}{2}x\right)+2\tan\left(\frac{1}{2}x\right)$	M1	Use of $\sec^2\alpha=1+\tan^2\alpha$
	$\int\left(\sec^2\left(\frac{1}{2}x\right)+2\tan\left(\frac{1}{2}x\right)\right)\mathrm{d}x=2\tan\left(\frac{1}{2}x\right)+2\ln\left(\sec\frac{1}{2}x\right)\times 2$	M1 A1	M1 for attempt to integrate ($k\tan\theta$ or $k\ln\sec\theta$) A1 for all correct
	$\int_0^{\frac{\pi}{2}}(\ldots)\,\mathrm{d}x=2\tan\frac{\pi}{4}+4\ln\sec\frac{\pi}{4}-(0)$	M1	Use of limits $\frac{\pi}{4}$ seen (provided some int. attempt)
	$= 2 + 4\ln\sqrt{2}$ **= 2 + ln4**	A1 A1 **(7)**	a = 2 b = 4 (Accept 2ln2) A1A1 dep. on 4th M only
3. (a)	$k, kp, kpq;\ kp^2q,\ kp^2q^2,\ kp^3q^2$	M1 A2/1/0 (3)	M1 for 1st 3 terms A2/1/0 (-1 eeoo) for next 3
(b)	**[Need one line clearly showing factorisation or split]** Identify: $k+kpq+kp^2q^2\ \ldots$ is GP with $a = k, r = pq$	M1A1	M1 for splitting into 2 series A1 for 1st a and r
	Identify: $kp+kp^2q+kp(pq)^2\ldots$ is GP with $a = kp, r = pq$	M1A1	M1 for identifying 2nd GP A1 for 2nd a and r
	$S_{2n}=\frac{k\left(1-(pq)^n\right)}{1-pq}+\frac{kp\left(1-(pq)^n\right)}{1-pq}$	M1	Use of Sn formula twice. One correct ft their a & r
	$=\frac{k(1+p)\left(1-(pq)^n\right)}{1-pq}$	A1cso (6)	
(c)	$\sum_1^{\infty}=6+6\times\left(\frac{4}{3}\right)+6\times\left(\frac{4}{3}\right)\times\left(\frac{3}{5}\right)+\ldots$ i.e. k = 6, $p=\frac{4}{3}, q=\frac{3}{5}$	B1	Identify link with above and values for k, p and q
	$r = pq = \frac{4}{5}$ ($r<1\ \therefore\ S_\infty$ formula can be used)	M1	Attempt to find r. (S+ for noting r <1 etc)
	$S_\infty=\frac{k(1+p)}{1-pq}=\frac{6\times\frac{7}{3}}{1-\frac{4}{5}},\quad=\frac{210}{3}$ = **70**	A1,A1 (4) **(13)**	A1 for an expression can be in k, p or q. ft their values A1 for 70

Question	Scheme	Marks	Notes
4. **(a)**	$2y = 2\sin t\cos t = \sin 2t$	M1	Use of sin2*t*
	$2x = 2\cos^2 t \quad \Rightarrow 2x-1 = 2\cos^2 t - 1 = \cos 2t$	M1	Use of cos2*t*
	$\left(2x-1\right)^2 + \left(2y\right)^2 = 1$	M1	Successfully eliminating *t* and eqn. for circle
	$\left(x-\tfrac{1}{2}\right)^2 + y^2 = \left(\tfrac{1}{2}\right)^2$ so centre $(\tfrac{1}{2}, 0)$, $r = \tfrac{1}{2}$	A1A1 (5)	A1 for centre A1 for radius
(b)	Area of $R = \cos^2\alpha \times \sin\alpha\cos\alpha = \cos^3\alpha\sin\alpha$	B1(1)	Some evidence of *xy* leading to given result
(c)	$\dfrac{\mathrm{d}A}{\mathrm{d}\alpha} = \cos\alpha\cos^3\alpha - 3\cos^2\alpha\sin^2\alpha$	M1A1	M1 for use of product rule
	$\dfrac{\mathrm{d}A}{\mathrm{d}\alpha} = 0 \Rightarrow \cos^2\alpha\left(\cos^2\alpha - 3\sin^2\alpha\right) = 0$	M1	M1 for setting derivative =0 and attempting to solve
	$\cos^2\alpha = 0 \Rightarrow \left[\alpha = \tfrac{\pi}{2}\right]$ or $\tan^2\alpha = \dfrac{1}{3} \Rightarrow \alpha = \dfrac{\pi}{6}$ (or 30^o)	A1 A1	A1 for "trig" =..A1 for α = .. Can ignore $\alpha = \tfrac{\pi}{2}$ but consider for S+
	$A'' = 2\sin\alpha\cos\alpha(3 - 8\cos^2\alpha)$ and show <0 for $\alpha = \dfrac{\pi}{6}$ or argument based on $\alpha = \tfrac{\pi}{2}$ gives min so this is max	M1	Some check that this value of α gives a max
	Maximum area is $\dfrac{3\sqrt{3}}{16}$ (o.e.)	B1 (7) **(13)**	Single fraction with rational denom
ALT (a)	$x^2 + y^2 = \cos^2 t$ or $\dfrac{y^2}{x} = \sin^2 t$	M1	Expression in *x* and *y* for $\cos^2 t$ or $\sin^2 t$
	$x^2 + y^2 = x$ or $\dfrac{y^2}{x} + x = 1$	M1	Equation in just *x* or *y*
	$\left(x-\tfrac{1}{2}\right)^2 + y^2 = \tfrac{1}{4}$ Then as in scheme	M1	An attempt to complete the square

<u>Marks for Style Clarity and Presentation (up to a max of 7 marks)</u>

<u>S1</u> For a fully correct and succinct solution to questions 1 or 2 **<u>or</u>** (*n* – 1) and some S+ in questions 3-7
<u>Or</u> for succinct solution of full marks on questions 3 – 7 but no S+ points seen
<u>S2</u> For a fully correct and succinct solution to questions 3 to 7 with some S+ points evident
<u>T1</u> For a good attempt at the whole paper (≥50% on each question)
Pick the best 3 S1/S2 scores to form the total

Question	Scheme	Marks	Notes
5. **(a)**	U is $\left(0,\ \frac{1}{2}\right)$	B1 (1)	For *y* coordinate
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\left(x^2-4\right)2x-\left(x^2-2\right)2x}{\left(x^2-4\right)^2},=\frac{-4x}{\left(x^2-4\right)^2}$	M1, A1	M1 for attempt to diff. (Two parts and one correct) Wrong formula used is M0 A1 when num. simplified
	Gradient of normal at P = $\frac{\left(a^2-4\right)^2}{4a}$	M1	Use of perpendicular gradient rule and *x* = *a*
	Equation of normal: $y-\frac{a^2-2}{a^2-4}=\frac{\left(a^2-4\right)^2}{4a}\left(x-a\right)$	M1	Attempt at eqn of normal can ft their changed grad
	x = 0 gives $y=\frac{a^2-2}{a^2-4}-\frac{\left(a^2-4\right)^2}{4}$ **(*)**	M1 A1cso (6)	M1 clear use of *x* = 0 in norm A1 for no incorrect working seen
(c)(i)	**No use of circle is 0/5 for (i)** Centre is at (0, k) [where k is y-coord from part (b)] Radius = y coord of their centre – 0.5	B1 B1	May be implied by a sketch radius touches at *U*
	Radius to P = $\sqrt{a^2+\left(k-\frac{a^2-2}{a^2-4}\right)^2}$ or $\sqrt{a^2+\frac{\left(a^2-4\right)^4}{16}}$	M1	Expression for radius from centre to *P*
	From (b) and k - 0.5: $\left[\frac{a^2-2}{a^2-4}-\frac{1}{2}-\frac{\left(a^2-4\right)^2}{4}\right]^2=a^2+\frac{\left(a^2-4\right)^4}{16}$	M1	For attempt at a suitable equation in *a* NB r^2 = LHS implies B1B1
	$\left[\frac{a^2}{2(a^2-4)}-\frac{(a^2-4)^2}{4}\right]^2=a^2+\frac{(a^2-4)^4}{16}$ **(*)**	A1cso (5)	
(ii)	$\frac{a^4}{4(a^2-4)^2}-\frac{a^2(a^2-4)}{4}+\cancel{\frac{(a^2-4)^4}{16}}=a^2+\cancel{\frac{(a^2-4)^4}{16}}$		[When cancel a^2 and consider *a* = 0 for S+]
	$\frac{a^2}{4(a^2-4)^2}=1+\frac{a^2-4}{4}$ $\left\{=\frac{4+a^2-4}{4}\right\}$	M1	Remove $\frac{\left(a^2-4\right)^2}{16}$ and cancel a^2
	$\left(a^2-4\right)^2=1$ **(*)**	A1cso	
(iii)	$a^2-4=\pm1$ so $a=\pm\sqrt{3}$ or $\pm\sqrt{5}$	A1	For $a^2=5$ or better, $\sqrt{3}$ can be ignored and ± Dependent on 3rd M1 [S+ for reason to reject $\sqrt{3}$]
	$k=\frac{5-2}{1}-\frac{1^2}{4}=\frac{11}{4}$ so centre is $\underline{(0,\frac{11}{4})\text{ rad is }\frac{9}{4}}$	A1A1 (5) **(17)**	A1 for centre, A1 for radius (Dependent on 3rd M1) [May imply some Bs]
	Allow them to start at (ii) but 3rd M1 is critical		

Question	Scheme	Marks	Notes
6. **(a)**	$\overrightarrow{PR}=\begin{pmatrix}13-5t--7\\-3+3t-2\\-8+4t-7\end{pmatrix}=\begin{pmatrix}20-5t\\-5+3t\\-15+4t\end{pmatrix}$	M1 A1	Attempt vector *PR*
	$\overrightarrow{PR}\bullet\begin{pmatrix}-5\\3\\4\end{pmatrix}=0\Rightarrow-100+25t-15+9t-60+16t=0$	M1	Attempt suitable scalar product
	$50t=175\Rightarrow t=\frac{7}{2}$	A1	
	If X is midpoint of PP' then $\overrightarrow{OP'}=\overrightarrow{OP}+2\overrightarrow{PX}$	M1	Strategy using known vectors
	$\overrightarrow{OP'}=\begin{pmatrix}-7\\2\\7\end{pmatrix}+2\begin{pmatrix}\frac{5}{2}\\\frac{11}{2}\\-1\end{pmatrix}=\begin{pmatrix}-2\\13\\5\end{pmatrix}$	A1 (6)	NB X is $\left(-\frac{9}{2},\frac{15}{2},6\right)$
(b)	Let t = 4 then can see A lies on L	B1 (1)	Showing t = 4 works
(c)	$\overrightarrow{AP}=\begin{pmatrix}0\\-7\\-1\end{pmatrix},\ \overrightarrow{AP'}=\begin{pmatrix}5\\4\\-3\end{pmatrix}\Rightarrow\overrightarrow{AP}\bullet\overrightarrow{AP'}=\frac{0-28+3}{\sqrt{50}\sqrt{50}}$ = - 0.5	M1 M1	Attempt suitable vectors($\pm$) Attempt suitable scalar product ($\pm$)
	So PAP' = 120^o	A1cso (3)	No incorrect working seen
(d)	$\lvert PP'\rvert$ = $\sqrt{5^2+11^2+(-2)^2}=\sqrt{150}\left[=5\sqrt{6}\right]$	B1	Attempt $\lvert PP'\rvert$ (oe) or use sin60
	Area = $\frac{1}{2}AB\times PP'=50\sqrt{3}\Rightarrow AB=10\sqrt{2}$ or $2\sqrt{50}$ o.e.	M1A1	M1 for attempt at equation giving length of *AB*
	$\lvert AX\rvert=\frac{1}{2}\sqrt{50}$ so $\overrightarrow{AB}=4\overrightarrow{AX}$ or when t = 2 in equation of L	M1	Strategy for finding *B*
	$\overrightarrow{OB}=\begin{pmatrix}3\\3\\0\end{pmatrix}$ $\left[\text{ignore } t=6\rightarrow\begin{pmatrix}-17\\15\\16\end{pmatrix}\right]$	A1 (5)	
(e)	$\overrightarrow{AP}=\begin{pmatrix}0\\-7\\-1\end{pmatrix},\ \overrightarrow{PB}=\begin{pmatrix}10\\1\\-7\end{pmatrix}\Rightarrow\overrightarrow{AP}\bullet\overrightarrow{PB}=0$ so angle is 90^o **(*)**	M1 A1cso (2)	Full method to find angle
(f)	Since *APB* is right angle *AB* is a diameter	M1	Using angle in semicircle theorem (S+ for mentioning)
	So centre is at midpoint $\frac{1}{2}\left[\begin{pmatrix}-7\\9\\8\end{pmatrix}+\begin{pmatrix}3\\3\\0\end{pmatrix}\right]=\begin{pmatrix}-2\\6\\4\end{pmatrix}$ or (-2,6,4)	A1 (2) **(19)**	
ALT (c)(d)	Finding $\overrightarrow{AP}$ and $\overrightarrow{AP'}$	M1	May show *PAB* = 60
	$\lvert\overrightarrow{AP}\rvert=\lvert\overrightarrow{AP'}\rvert=\sqrt{50}$	M1	
		B1	B1 for $\lvert PP'\rvert$ from (d)

	$\left\|\overrightarrow{PP'}\right\| = \sqrt{150}$; $\sin(PAX) = \dfrac{\frac{1}{2}\sqrt{150}}{\sqrt{50}} = \dfrac{\sqrt{3}}{2} \Rightarrow PAP' = 120^o$	A1cso	Then as in main scheme
7. **(a)**	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{(3-x)2x + (x^2-5)}{(3-x)^2}$ or $y = -3 - x + \dfrac{4}{3-x} \Rightarrow y' = -1 + \dfrac{4}{(3-x)^2}$	M1 A1	M1 for an attempt to differentiate A1 any correct ver.
	$y' = 0 \Rightarrow x = 1$ or 5	M1	Find stat points
	A is (1, -2) and *B* is (5, -10)	A1A1 (5)	Full coords
(b) **(i)**	Horizontal translation 3 to left so *p* = 3	B1	
	-2 + *q* = -(*q* - 10) , so *q* = 6	M1A1	M1 for a correct identifiable strategy for *b* e.g. eqn for *q*
(ii)	*D* is (2, 4)	B1B1 (5)	(B1, B1)
(c) **(i)**	$y = \dfrac{x^2-5}{3-x} \Rightarrow 3y - xy = x^2 - 5$	M1	Set *y* = and 1st step
	$3y + 5 = x^2 + yx \Rightarrow \left(x + \frac{y}{2}\right)^2 = 3y + 5 + \dfrac{y^2}{4}$	M1	Isolate *x*'s or set up as 3TQ and attempt to solve for *x*
	$x + \dfrac{y}{2} = \pm\dfrac{\sqrt{y^2+12y+20}}{2}$ o.e. (Accept +, $-$or $\underline{+}$)	A1	[S+ for reason for choosing $-$] Must choose –
	$x = \dfrac{-y - \sqrt{y^2+12y+20}}{2}$ $\left[\text{so } \mathrm{m}^{-1}(x) = \dfrac{-x - \sqrt{x^2+12x+20}}{2}\right]$	A1 (4)	
(ii)	Domain is range of m(*x*) i.e. $(x \in \mathbb{R},)\, x \geq -2$	B1 (1)	
(iii)	If $\mathrm{m}(t) = \mathrm{m}^{-1}(t)$ then m(*x*) intersects with *y* = *x*	M1	Suitable strategy leading to an eqn for *t*. ft their m^{-1}
	$\dfrac{t^2-5}{3-t} = t$	A1	A correct quadratic equation
	$2t^2 - 3t - 5(= 0)$	M1	Solving correct 3TQ
	$(2t-5)(t+1) = 0$	A1	correct factors (A1)
	***t* = - 1** (or 2.5)	A1 (5)	(-1 only) [S+ for reason]
	Can't be 2.5 since not in domain for m(*x*)	**(20)**	

Further copies of this publication are available from
Edexcel Publications, Adamsway, Mansfield, Notts, NG18 4FN

Telephone 01623 467467
Fax 01623 450481
Email publication.orders@edexcel.com
Order Code UA028410
June 2011

For more information on Edexcel qualifications, please visit
www.edexcel.com/quals

Pearson Education Limited. Registered company number 872828
with its registered office at Edinburgh Gate, Harlow, Essex CM20 2JE

Llywodraeth Cynulliad Cymru
Welsh Assembly Government

Rewarding Learning